Exam. Code : 211001
Subject Code : 4849

M.Sc. Mathematics 1st Semester

MECHANICS—I

Paper—MATH-554

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. All questions carry equal marks.

SECTION—A

1. (a) Obtain the radial and transverse components of velocity and acceleration of the motion of a particle in plane.

(b) The points (a, 2a, –a), (–a, –a, a), (a, a, a) of a rigid body have instantaneous velocity $\left(\frac{\sqrt{3}v}{2}, 0, \frac{\sqrt{3}v}{2}\right)$, $\left(\frac{-v}{\sqrt{3}}, 0, \frac{-v}{\sqrt{3}}\right)$, $\left(0, \frac{-v}{\sqrt{3}}, \frac{v}{\sqrt{3}}\right)$.

Show that the body has the line through the origin having direction cosines $\left[\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right]$ as instantaneous axis of rotation and that the magnitude of the angular velocity is $\frac{v}{2a}$.

2. (a) A rigid body s has a spin w and a particle A of S has velocity $\vec{v}$. Show that every particle P of S with velocity vector parallel to $\vec{w}$ lies on the line $\overrightarrow{AP} = (\vec{w} \times \vec{v}) w^2 + \mu \vec{w}$, μ is arbitrary scalar.

(b) Prove that :

$$\left.\frac{d\vec{r}}{dt}\right|_F = \left.\frac{\partial \vec{r}}{\partial t}\right|_M + \vec{w} \times \vec{r},$$ where the symbols have their usual meaning, use it to find the velocity components of a point in spherical polar co-ordinates.

SECTION—B

3. (a) Explain rectilinear particle motion with respect to uniform accelerated motion and resisted motion.

(b) A particle of mass m is placed on a horizontal board which is made to execute vertical simple harmonic oscillations of period T and amplitude a. If $a < (gT^2/4\pi^2)$, show that the particle does not lose contact with the board at any time.

4. (a) A fixed wire is in the shape of the cardioid $r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point $r = 0$ of the cardioid by an elastic string of natural length a and modulus 4 mg. If the particle is released from rest when the string is horizontal, show that $a\dot{\theta}^2 (1 + \cos\theta) - g\cos\theta\,(1 - \cos\theta) = 0$.

(b) Show that if the moment of the resultant force about the axis $\hat{a}$ is zero then the angular momentum $\hat{G} \cdot \vec{H}$ of the particle about the axis is constant.

SECTION—C

5. (a) Derive the differential equation of orbit of a particle moving under central force. Show that the inverse square law of force directed towards a fixed point always produces a conic type orbit.

(b) A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove with the usual notation $v^2 = \mu(2/r - 1/a)$, $h^2 = \mu a(1 - e^2)$ when the particle is at one end of a minor axis, its velocity is doubled. Prove that the new path is a hyperbola of eccentricity $(9 - 8e^2)^{1/2}$.

6. (a) Two gravitating particles of masses m and M move under the force of their mutual attraction. Show that the centre of mass of the two particles moves with constant velocity, and that if $\vec{r}$ is the position vector of m relative M, $\ddot{\vec{r}} = -\gamma(M + m)\frac{\hat{r}}{r^3}$, where γ is the gravitational constant. If the orbit of m relative to M is a circle of radius a described with velocity v, show that $v = [\gamma(M + m)/a]^{1/2}$.

(b) Write a note on elliptic harmonic motion.

SECTION—D

7. (a) Determine the moment of inertia of the distribution about the axis through O having direction cosines $[\lambda, \mu, \nu]$ in terms of there D.Cs. and A, B, ...F.

   (b) Prove that there exists three principal directions at a point of a rigid body which are real and mutually orthogonal.

8. (a) State and prove the necessary and sufficient conditions for the two systems to be equimomental.

   (b) Show that in two-dimensional mass distributions, the principal directions with usual notations are given by $\tan 2\alpha = \frac{2F}{B - A}$.