(b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, prove that : **8**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 2u\left(1-4\sin^2 u\right)$$

5. (a) If $x=u^2-v$, $y=v^2-u$; find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$. **8**

(b) Expand $\tan^{-1}\frac{y}{x}$ in the neighbourhood of $(1, 1)$ by Taylor's theorem as far as terms of second degree. **8**

Unit III

6. State Schwarz's Theorem. For the function :

$$f(x, y)=\begin{cases}\frac{1}{4}\left(x^2+y^2\right)\log\left(x^2+y^2\right) & , (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$$

Show that $f_{xy}=f_{yx}$ for all x, y but the conditions of Schwarz's theorem are not satisfied. **16**

Roll No. .... Exam Code : J-21

Subject Code—52466

B. Sc. EXAMINATION

(Main/Re-appear) (Batch 2018 Onwards)

(Third Semester)

MATHEMATICS

CML-306 (Course V)

Advanced Calculus

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt any *Five* questions. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Test the function for continuity : **2½**

$$f(x)=\begin{cases}(x-a)\sin\frac{1}{x-a} & \text{when } x\neq a\\ 0 & \text{when } x=a\end{cases}$$

(b) If $y = \frac{u}{u-1}$ and $u = x - \frac{1}{x}$; find $\frac{dy}{dx}$. **2½**

(c) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, then show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$ **2½**

(d) Find $\frac{du}{dt}$, when $u = xy^2 + x^2 y$, $x = at^2$, $y = 2at$. **2½**

(e) Evaluate the indeterminate form : **2**

$$\lim_{x \to 0+} \frac{\operatorname{cosec} x}{\log x}$$

(f) Evaluate the integral in terms of Beta function : **2**

$$\int_0^1 x^m \left(1 - x^2\right)^n dx, \ m > -1, n > -1$$

(g) Evaluate the integral : **2**

$$\iint_R x\,dx\,dy, \text{ where}$$

$$R = \left\{(x, y) : 0 \le x \le a, 0 \le y \le x\right\}.$$

Unit I

2. (a) Prove that the function defined by $f(x) = \sin\frac{1}{x}$, $x \in R^t$ is continuous but not uniformly continuous on R^t. **8**

(b) Verify Rolle's Theorem for $f(x) = \log\left(x^2 + 2\right) - \log 3$ in [–1 , 1]. **8**

3. (a) Verify lagrange's mean value theorem for $f(x) = \log x$ in [1, e]. **8**

(b) Prove that : **8**

$$\lim_{x \to \infty}\left(1 + \frac{k}{x}\right)^x = e^k.$$

Unit II

4. (a) Show that the function f defined by :

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x, y) \ne (0, 0) \\ 0 \quad, & (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). **8**

9. (a) Prove that the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{4}{3}nabc.$ **8**

(b) Change the order of integration in $\int_0^a\int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ and hence evaluate the same. **8**

7. (a) Examine for maximum and minimum values the function : **8**

$$xy+\frac{a^3}{x}+\frac{a^3}{y},\ a>0$$

(b) Find the minimum value of the function $x^2+y^2+z^2$ subject to the condition $x+y+z=3a.$ **8**

Unit IV

8. (a) Evaluate :

$$\int_0^{n/2} \sin^n \theta d\theta$$

(b) Evaluate the integral :

$$\int_0^1\int_{\sqrt{y}}^1 dx\,dy$$

and sketch the region of integration. **8**